Exercise 21

Evaluate the integral.

$$\int_0^2 \left(\frac{4}{5}t^3 - \frac{3}{4}t^2 + \frac{2}{5}t\right) dt$$

Solution

According to part 2 of the fundamental theorem of calculus,

$$\int_a^b f(x)\,dx = F(b) - F(a),$$

where F is an antiderivative of f. Use the properties of integrals given at the bottom of page 385 to simplify the integral before using this theorem.

$$\begin{aligned}
\int_0^2 \left(\frac{4}{5}t^3 - \frac{3}{4}t^2 + \frac{2}{5}t\right) dt &= \int_0^2 \frac{4}{5}t^3\,dt - \int_0^2 \frac{3}{4}t^2\,dt + \int_0^2 \frac{2}{5}t\,dt \\
&= \frac{4}{5}\int_0^2 t^3\,dt - \frac{3}{4}\int_0^2 t^2\,dt + \frac{2}{5}\int_0^2 t\,dt \\
&= \frac{4}{5}\left(\frac{t^4}{4}\right)\Big|_0^2 - \frac{3}{4}\left(\frac{t^3}{3}\right)\Big|_0^2 + \frac{2}{5}\left(\frac{t^2}{2}\right)\Big|_0^2 \\
&= \frac{4}{5}\left(\frac{2^4}{4} - \frac{0^4}{4}\right) - \frac{3}{4}\left(\frac{2^3}{3} - \frac{0^3}{3}\right) + \frac{2}{5}\left(\frac{2^2}{2} - \frac{0^2}{2}\right) \\
&= \frac{4}{5}(4) - \frac{3}{4}\left(\frac{8}{3}\right) + \frac{2}{5}(2) \\
&= 2
\end{aligned}$$